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## The logic of thinking

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## INTRODUCTION

The point of this presentation is to undermine a few generally accepted ideas about logic by showing that modern standard logic, in the form in which it has crystallised since roughly 1900, is far from the one and only possible immutable logic it has been taken to be. On the contrary, a simple inspection of the foundations of logic quickly reveals ways of producing a large number of logically sound alternatives that deviate in often interesting ways from standard logic. We will demonstrate this for predicate calculus. Propositional calculus will be left alone, even though there, too, one sees interesting and fruitful ways of varying upon the standard theme.<sup>1</sup>

The interest of this new approach lies in the fact that modern standard logic may be of great use to mathematics but corresponds rather badly with intuitive judgements of truth and falsity produced by ordinary people who express their thoughts in language. By contrast, traditional predicate calculus, usually called ‘Aristotelian’, scores significantly better in this respect, but it has the serious drawback of suffering from one fatal logical

<sup>1</sup> See, for example, Seuren et al. (2001) for a three-valued propositional calculus serving the purposes of presupposition theory and discourse-dependency.

defect, known as ‘undue existential import’, and a few minor ones that are not fatal. For that reason it was unceremoniously dumped around the year 1900 and replaced with modern standard predicate calculus.

This placed philosophers of language and semanticists of natural language in an uncomfortable dilemma. In the beginning they were browbeaten by logicians like Bertrand Russell, who called natural language names. Language, it was said, is vague, ambiguous and generally unreliable for logical or scientific purposes. Soon, however, it became clear that natural language is of significant interest to a logician and deserves further study from this point of view. Yet even though after 1970, under the influence of the American logician Richard Montague, research to that effect became very fashionable, and to some extent also fruitful, in the context of so-called *formal* or *model-theoretic semantics*, the discrepancies with natural intuitions remained. And as this fact was increasingly recognized, a new discipline, called *pragmatics*, appeared some thirty or so years ago, which attempted to bridge the gap between formal logical analysis and natural intuitions by an appeal to general principles of rational communicative behaviour of people engaging in linguistic interaction.

However, in so far as the applicability of logic to language is concerned, these attempts have met with less than complete success. At least in this respect, pragmatics keeps being plagued by a lack of precision, which makes it virtually unfalsifiable, and thus scientifically weak. One of the things pragmaticists are trying to achieve is to restore traditional Aristotelian predicate calculus, not, however, as a sound system of logic but as a communicatively functional manifestation of modern standard logic. One can, of course, take different views about that. But when it turns out, as it now does, to be possible to produce a whole gamut of logically sound alternative logics, including varieties that incorporate the logical properties of Aristotelian predicate calculus but do not suffer from undue existential import, then clearly an entirely new perspective arises with regard to the way the logical properties of natural language are to be looked at. In this paper, various ways will be shown to restore those elements in the Aristotelian tradition of predicate calculus that have offended modern logicians to their rightful place in the logic of language and thus, in important respects, also in the logic of thinking. This logic will then be a well-defined logico-semantic system, no longer in need of a pragmatic ersatz. Not only will the uncomfortable dilemma that semanticists and philosophers of language had to face thus be removed, we also get a number of interesting new insights into the nature and the foundations of logic into the bargain.

As a matter of historical interest, it will also be shown that the traditional

system of predicate calculus, though called 'Aristotelian', appears to differ significantly from what is found in Aristotle's own texts. Not only is the logic found in Aristotle's own writings incomplete, in that it allows for further theorems that he himself apparently failed to see but were supplied by later commentators and interpreters, there is also a highly interesting inconsistency.

Meticulous reading reveals that Aristotle did not propose the so-called Conversions (equivalence of 'not-all-not' with 'some', and of 'not-some-not' with 'all'), but only one-way entailments from 'all-not' to 'not some' and from 'some' to 'not-all-not', leaving out their converses. This, as will be shown, saves the system from undue existential import and thus takes away the central objection to this logic. Yet, at the same time, he assigned existential import to 'not-all', which, under his own concept of standard bivalent negation, is inconsistent with a rejection of the Conversions.

The later Aristotelian tradition accepted both the Conversions and existential import for 'not-all', but made the system unsound by accepting existential import for 'all' as well. This was rectified in modern logic, which accepted the Conversions as well as existential import for 'not-all', but denied existential import to 'all'. However, the fact that a sound logic (with standard bivalent negation) is possible when the Conversions are rejected and existential import is denied to 'not-all', has been almost universally overlooked by logicians of all ages (see note 5 below). In fact, any sound system of predicate calculus under standard bivalent negation must have either both the Conversions and existential import for 'not-all', or neither the Conversions nor existential import for 'not-all': these two elements either occur together or they are both absent. One can only be amazed at finding that this crucial element in the history and the critique of logic has remained so underexposed.

#### SOME BACKGROUND

So as to have a more precise view of these matters we must have a quick look at some historical backgrounds. Until the end of the 19th century logic was commonly defined as 'the study of the principles of correct reasoning' or 'the discipline of correct reasoning'. The Oxford philosopher Thomas Fowler, for example, wrote (Fowler 1892:2-6):

The more detailed consideration of ... Thoughts or the results of Thinking becomes the subject of a science with a distinct name, Logic, which is thus a subordinate branch of the wider science, Psychology. ... It is the province of Logic

to distinguish correct from incorrect thoughts. ... Logic may therefore be defined as the science of the conditions on which correct thoughts depend, and the art of attaining to correct and avoiding incorrect thoughts. ... Logic is concerned with the *products* or results rather than with the *process* of thought, i.e. with *thoughts* rather than with *thinking*.

Similar statements are found in virtually all logic textbooks of that period. After 1900, however, changes are beginning to occur, slowly at first but then, especially after the 1920s, much faster, until the 19th-century view of logic fades away entirely during the 1960s, with Copi (1961) as one rare late representative.

What, then, happened around 1900? Two things. First, logic went *mathematical*. This means that logic was interpreted as a formal calculus of the kind current in mathematics. (One should realize that since about 1850 mathematics itself had been going through a period of rapid changes, in that it became both more abstract and more general.) Secondly, logic was dissociated from thinking and human cognition, which was either given a subordinate role or eliminated altogether. Let us call this the *depsychologisation* of logic.

Both developments were initiated by the Czech philosopher Bernard Bolzano (1781-1848), with his voluminous *Wissenschaftslehre* of 1837. Largely forgotten till the 1870s, it then had a profound influence on both Franz Brentano and Gottlob Frege (1848-1925). The latter inspired Bertrand Russell (1872-1970), who became the public mouthpiece of the new movement, to which he also made some important contributions. In his well-known book *The Principles of Mathematics* of 1903 Russell reduced logical consequences (entailments) to some kind of metaphysical necessity, without the intervention of the human mind, which, if anything, was taken to be a mere transmitting device. Thus we read (Russell 1903:33):

In the discussion of inference, it is common to permit the intrusion of a psychological element, and to consider our acquisition of new knowledge by its means. But it is plain that where we validly infer one proposition from another, we do so in virtue of a relation which holds between the two propositions whether we perceive it or not: the mind, in fact, is as purely receptive in inference as common sense supposes it to be in perception of sensible objects.

The combination of mathematicisation and depsychologisation became a hallmark not only of logic but also of Anglosaxon analytic philosophy and of large sections of the human sciences, including even psychology. In linguistics the influence of this development was strongly felt as well (see

Seuren 2002 for further comment). In both linguistics and logic, mathematicisation has proved to be very beneficial. Depsychologisation, on the other hand, has, in this author's view, led to serious derailments. The depsychologisation of logic made people blind to the fact that logical entailment relations are determined by the meanings of the logical constants in the logical language at hand. (Russell and Wittgenstein, with their profound confusion of basic issues in logic, have a lot to answer for.) As regards logic, the analyses offered in the present paper may well be seen as a reversal to the 19th century, when the mind was still fashionable, while full advantage is taken of the technical (mathematical) developments achieved during the 20th century.

Every logic must be expressed in a language, which may be a strongly stylised, or (in Quinean terminology) 'regimented', form of a natural language or a formal language designed strictly for the purpose at hand. In such a language, a few *logical constants* are singled out, which enable one to compute entailments instead of deriving them intuitively. In the simplest form of predicate calculus the logical constants are *all*, *some* (that is, *at least one*) and *not*. The remaining elements that may occur in the sentences at issue are represented as variables (usually F, G, etc.), which take lexical predicates as values. In propositional calculus, the standard logical constants are, again, *not*, and also *and*, *or* and if-then, and the variables stand for propositions. In modal logic, the modal predicates are added as logical constants.

When we speak of (*logical*) *entailment*, we mean a logically necessary consequence resulting from the meanings of the logical constants involved. **To say that a sentence *A* logically entails a sentence *B*, or, (in formal notation,  $A \vdash B$ ), means that whenever *A* is true, *B* must of necessity also be true, on account of the meanings of the logical constants involved in *A* and *B*.** (*Logical equivalence*, moreover, is logical entailment in both directions: '*A* is logically equivalent with *B*', or ' $A \equiv B$ ' means that  $A \vdash B$  and  $B \vdash A$ . Logic is thus the formal calculus of entailments.

#### ARISTOTELIAN SYLLOGISTIC

Traditional logic, as handed down through the ages, was neither mathematical nor antipsychological. It consisted mainly of predicate calculus and syllogistic, both largely derived from Aristotle. We will discuss predicate calculus in a moment. Syllogistic was the doctrine of setting up a logical argument consisting of three elements, a *major (Ma)*, a *minor (Mi)*, and a *conclusion (C)*. An Aristotelian syllogism is of the form:

If  $Ma$  and  $Mi$ , then  $C$

where *if-then* and *and* are logical constants, and  $Ma$ ,  $Mi$  and  $C$  are variables standing for propositions (statements) of certain regimented types. For example (in terms of an inference schema instead of the original Aristotelian implicational schema):

- (1) Major: No human is immortal.  
 Minor: All Englishmen are humans.  
 Conclusion: No Englishman is immortal.

Or, in schematic form:

- (2) Major: No F is G (type E)  
 Minor: All H is F (type A)  
 Conclusion: No H is G (type E)

Major, minor and conclusion were always of a certain regimented type, taken from Aristotelian predicate calculus. The tradition has four such types:

- (3) type A: All F is G (or equivalently: No F is not-G)  
 type I: Some F is G (or equivalently: Not all F is not-G)  
 type E: No F is G (or equivalently: All F is not-G)  
 type O: Not all F is G (or equivalently: Some F is not-G)

The names **A**, **I**, **E** and **O** were introduced by the early Christian philosopher Boethius ( $\pm$  500 AD), who named the types **A** and **I** after the first two vowels of the Latin word *affirmo* ('I affirm'), and the types **E** and **O** after the vowels of the Latin word *nego* ('I deny'). As Boethius had a decisive influence on Medieval logic, his system of representation became standardly accepted during the Middle Ages and later. (One notes that each sentence type has an equivalent alternative translation, based on Conversion. Although Aristotle himself did not explicitly accept these equivalences (see below), the tradition has almost universally accepted them, up until the present day. Since the question is of minor relevance in the context of syllogistic proper, we can ignore it here. It is taken up below in the context of predicate calculus.)

In medieval times, the syllogistic reasoning schemata made up of these four sentence types were each given a gibberish Latin name that served as a memory aid for young students. The syllogistic schema shown in (2), for example, was called **CELARENT**, as it is built up from the sentence types

E, A and E. Other such names were BARBARA, DARII, FERIO, etc. Aristotle distinguished 14 such schemata. Five more were added during the Middle Ages. Whoever wished to master the technique of syllogistic reasoning had to memorize all these schemata, which is where the medieval fancy names came in as convenient props.

Around 1840, however, the Yorkshireman George Boole (1815-1864) discovered that the syllogistic schemata did not have to be memorized but were all derivable, in a very simple way, from the abstract algebra that he had developed, now standardly called Boolean algebra, which served as a formal basis for set theory. Not that Boole was the first to look for a mathematical basis of syllogisms. After a first start made by Leibniz in the late 17th century, we find the Swiss mathematician Leonhard Euler (1707-1783) carrying on in this vein. He 'translated' the four sentence types in terms of sets, as shown in fig. 1.

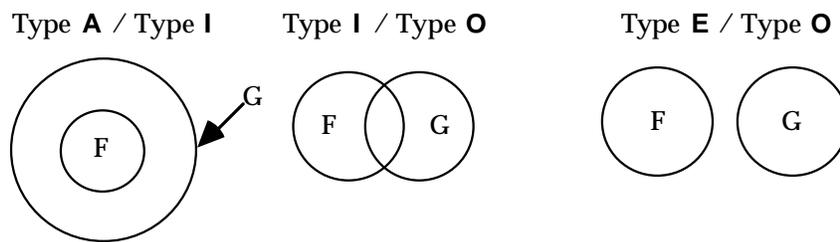


Figure 1. Euler-diagrams for the sentence types A, I, E and O.

Yet a proof theory was still lacking. Fig. 2 shows how CELARENT can be read from Euler diagrams. It is now immediately clear that if no F is G (type E) and all H is F (type A), then no H can be G (type E).

Around 1880 John Venn improved on the Euler diagrams by introducing the now famous Venn diagrams.<sup>2</sup> The formal proof, however, is ultimately delivered in terms of the underlying Boolean algebra. This effectively destroyed Aristotelian syllogistic, which was replaced with a mathematical, i.e. Boolean, proof theory.

<sup>2</sup> For a full exposé of the Venn diagram system, see e.g. Ambrose & Lazerowitz (1948:218-229) or Kneale & Kneale (1962:67-81).

**CELARENT:**

No F is G

All H is F

So: No H is G

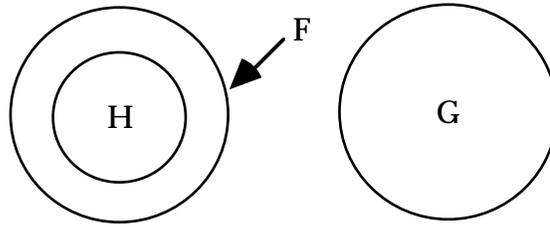


Figure 2. CELARENT in Euler-diagrams.

ARISTOTELIAN PREDICATE CALCULUS IN ITS ORIGINAL FORM<sup>3</sup>

The old syllogistic was based on Aristotelian predicate calculus (APC), as set out originally in Aristotle's *De Interpretatione* and in the first few pages of his *Prior Analytics*. This was subsequently developed by ancient commentators and logicians, in particular the Greek Ammonius and his younger Roman contemporary Boethius (5th-6th century AD). As has been said, it was Boethius who provided the formalization of Aristotelian predicate logic, in terms of the sentence types **A**, **I**, **E** and **O**, that became the standard during the Middle Ages and later, and that generations of students have lived with. We will summarize briefly what Aristotle's original predicate logic may be taken to have amounted to and how it relates to the system of logic that was known and practised during the Middle Ages and later and has come to us as traditional predicate logic.

It must be observed first that *De Interpretatione* is not primarily about predicate logic but about the structure and semantics of statements (De Rijk 2002:190, 352), in particular, it must be added, with regard to the function of negation. It is in this wider context that what was soon set apart as the predicate logic proper is developed and presented. It should, therefore, come as no surprise that *De Interpretatione* contains many passages that are not directly related to the actual machinery of predicate logic but deal with questions in a broader perspective. Thus one finds exposés on lexical meaning, on the distinction between polar (yes-no) questions and  $_{WH}$ -

<sup>3</sup> I am indebted to Lambert de Rijk for allowing me to profit from his vast knowledge of Aristotle. The fact that I have not followed his advice in every respect is entirely due to my own conceit.

questions, on the metaphysics of truth with regard to future statements (the famous chapter 9 on the sea battle that will or will not take place tomorrow), on modalities in relation to internal and external negation, and other topics as well.

Reading *De Interpretatione* is not an easy matter. The text is exploratory more than expository, as it reflects Aristotle's progress in the development of structural, semantic and logical notions, which were totally unknown in his day, and thus even more exotic and alien to ordinary minds than they still are today. In fact, the precise make-up of the text is sometimes uncertain in small but highly significant details such as the presence or absence of a negation word in an unexpected position. Aristotle's terminology is still in the making and his logical system of entailment schemata incomplete; his analyses are often still uncertain, and his powers of formalisation or of graphical representation limited. Little wonder that one finds some unsatisfactory analyses and even, apparently, a few inconsistencies, given the text that is available.

Our concern here, however, is not so much with the wider issues but, rather, with the bare machinery of predicate logic. A few general notions must be clarified first. A *statement* (also often called 'sentence' or 'proposition') is the expression of the mental process of assigning a property or attribute to an entity or class of entities (*PrAn* 24a16). The entity or entities are the *hypokeímenon* (the Latin terms are, variously, *subiectum*, *suppositum*, *substratum*). The property or attribute is the *symbebékós* (Latin *accidens*). The corresponding sentence parts are mostly called *ónoma* ('name') for the subject term, and *rhêma* ('verb') or *katégoroúmenon* (Latin *praedicatum*) for what we will call the clausal predicate (as opposed to lexical predicates). The modern notion of a predicate as a higher order expression denoting a class, to which lower order entities may or may not belong, is not found in Aristotle. For him the subject and the predicate of a sentence are terms on an equal footing. The thinking mind either 'unites' the mental representations that correspond to the subject and the (clausal) predicate, in which case an affirmative statement comes about, or it 'separates' them, producing a negative statement. However, 'in each and every case that which unifies [or separates] is the mind' (*De Anima* 430b5).

A 'name' in subject position is either a proper name or contains a general term (we would say 'common noun' or 'nominal predicate'), which may be applied to a number of individual entities. Common nouns may be negated, so that besides the common noun *man* we also have *not-man*. Such negated common nouns are not called simply 'names' by Aristotle. For lack of a better alternative, he calls them 'indefinite names' (*Int*

16a32). Likewise, verbal predicates can be negated, as in ‘not-walk’. These are also not called simply ‘verbs’, but again, for lack of a better term, ‘indefinite verbs’ because ‘they can equally well be truthfully predicated of anything whatsoever, whether existent or nonexistent’ (*Int* 16b15).

A statement may be built up out of two common nouns, whether negated or not. Such statements are also called ‘indefinite’. They are of the form (in word-by-word translation) ‘is-white-man’, and its negation ‘not-is-white-man’. Most (English) translations have ‘man is white’ and ‘man is not white’, respectively, to be taken in an ill-understood generic sense.

Given the possibility of negating both the subject term and the clausal predicate, there are four affirmative statement forms of this type: ‘man is white’, ‘not-man is white’, ‘man is not-white’, and ‘not-man is not-white’. All these may have the contradictory sentence negation: ‘man is not white’, ‘not-man is not white’, ‘man is not not-white’, and ‘not-man is not not-white’. It is clear from *Int* 21b27, that these sentences are treated as involving two conjoined *hypokeimena*, whatever is man and whatever is white, plus a predicate of being or non-being.<sup>4</sup>

This type of statement is highly problematic not only on account of its status in the Greek language, its grammatical structure and its semantics, but also because of its unclear logic (cp. Kneale & Kneale 1962:55). Large numbers of commentators have, through the centuries, filled volumes with comments and interpretations. Since, however, such ‘indefinite’ statements are not given a clear logic by Aristotle, his more logic-oriented commentators have excluded them from the machinery of predicate logic, a state of affairs that has remained so till the present day. We will, therefore, leave them out of account here.

Aristotle distinguishes singular from general statements. The former take proper names or definite referring descriptions as subject term, the latter involve some form of quantification. Since the distinction between

4 This is the reading defended by De Rijk (2002). He assigns great weight to the marked sentence-initial position of the Greek word *ésti* (‘is’), with concomitant accent on the first syllable. This features invites the substantive interpretation ‘has being’ or ‘is existent’, which makes this ‘is’ distinct from the bleached copula verb ‘is’, a distinction Aristotle himself insists on (*Int* 21a27). De Rijk maintains (despite *Int* 21b10, where Aristotle explicitly equates ‘a man walks’ and ‘a man is walking’) that Aristotle never uses the copulative ‘is’ in his analyses, but only the substantive ‘is’ that stands for ontological being. His reading (2002:259 & passim) of this type of statement is ‘Is: [(man&white)’s being]’, or ‘there is reality to the combination of humanhood and whiteness’ (see also Kneale & Kneale 1962:62-67). Although the question of how such structures are to be interpreted is of considerable philosophical interest, we shall leave it undiscussed here, as it has no direct bearing on the logical machinery proper.

contradictory (or external) and contrary (or internal) negation, about which more in a moment, does not apply to singular statements but only to universally quantified statements, the singular statements are not considered interesting from a logical point of view, and Aristotle subsequently devotes all his logical attention to quantified sentences. Among these he distinguishes three categories: universal, particular and 'indefinite' (*PrAn* 24a16). Again, the indefinites are discarded for the purposes of logic, given their unclear status, as has been illustrated above. This means that his predicate logic, and consequently his syllogistic, are built on two types, the *universal* (*kathólou*) and the *particular* (*en mérei*) statements. The universal statements receive most attention. They have the form 'all F is G' (or sometimes: 'G is predicated of all F') on the one hand, and 'no F is G' ('G is predicated of no F') on the other. Particular statements are given much less attention: the term *en mérei* occurs only once in *De Interpretatione*, at 23a17 (in the context of modal logic), but actual examples occur more frequently. Universal statements of the form 'all F is G' or 'no F is G' are repeatedly said to have 'F' as their subject term and '(is) G' as their clausal predicate term, while 'all' or 'no' provide the extra specification that the subject term is 'kathólou' (in modern terminology: under the universal quantifier; *Int* 20a13-4).

The sentence form 'some F is G' is a particular. Implicitly, the type 'not all F is G' is also taken to instantiate a particular statement form. Particular statements have so-called *existential import*: both 'some F is G' and 'not all F is G' entail the existence of at least one instantiation of F (see *Int* 20a23, the last sentence in the quote given below).

Great emphasis is placed on the function of sentence (or external) negation as an operator that generates *contradictories* (*antiphatikôs antikeimena*): a (nonambiguous) statement *A* and its negation *not-A* are contradictory because they can never be simultaneously true or simultaneously false: the truth of the one entails the falsity of the other, and vice versa. Thus, 'all F is G' and 'not all F is G' form a pair of contradictories, and likewise the pair 'some F is G' and its negation 'no F is G'. Moreover (though Aristotle remains silent on this point), two negations in immediate succession cancel each other out: 'not not all F is G' is equivalent with 'all F is G', and likewise for all other sentence types.

The contradiction-forming external negation, however, is matched by the internal clausal-predicate negation. The subject and clausal-predicate terms can themselves be negated, which makes them 'indefinite' in the sense indicated above. The following forms are, therefore, legitimate:

all/some F is G

not all/no F is G

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all/some not-F is G	not all/no not-F is G
all/some F is not-G	not all/no F is not-G
all/some not-F is not-G	not all/no not-F is not-G

Again, two successive predicate negations cancel each other out: *not-not-P* is logically equivalent with *P* for any predicate *P*. Despite these multiple possibilities, however, the negated *F*-term fails to play a role of any significance in the predicate logic that can be culled from *De Interpretatione* and *Prior Analytics*. In fact, the logic works only with the external sentential negation and the internal predicate negation of the *G*-term, i.e. the predicate.

‘All *F* is *G*’ and ‘no *F* is *G*’ are not contradictories, but *contraries* (*enantiai*), since they cannot be simultaneously true, though they can be simultaneously false. However, the contradictories of the two universal contraries, i.e. ‘some *F* is *G*’ and ‘not all *F* is *G*’, can be true at the same time (*Int* 17b25). This is, in fact, the default case: when it is said that some children are happy, it is natural, but not logically necessary, to infer that not all children are happy, that is, that some children are not happy.

Despite the at times excruciating opacity of Aristotle’s text, one infers from chapter 7 (17a32-18a12) of *De Interpretatione* that he draws a distinction between the logical properties of *singular* statements and those of *quantified* statements. For singular statements it makes no difference whether the negation is the external sentence negation or the internal clausal-predicate negation. In modern terms we can say that to assert that it is not the case that Callias is brave is the same as saying that Callias is not-brave, since in both cases the individual called ‘Callias’ is said not to be a member of the class of brave entities. In Aristotle’s own terms the same conclusion is reached. De Rijk quotes *De Anima* 430b1-4, where Aristotle states (in the context of singular statements):

Falsehood always involves a unification. For even if you assert that what is pale is not pale you have included not-pale in a unification. But it is equally possible to call all these cases ‘separation’.

In *quantified* statements, however, the difference between the external sentence negation and the internal clausal-predicate negation is essential, as has been shown.

One should note, in this context, that the modern notion of internal negation, that is, negation under a higher operator (small scope negation), was not available to Aristotle. Within the restricted terms of his analysis, the only two negations possible were external sentence negation generating a contradictory statement, and the internal clausal-predicate

negation, with different logical properties depending on the case at hand. However, the moment we adopt the modern analysis in terms of logical operators and variables, provided we stay within the standard Aristotelian axiom of strict bivalence, it no longer makes any difference whether the internal negation is treated as predicate negation or as a sentential operator over a sentential function containing a variable. Therefore, although we write systematically, for example, 'Some F is not-G', we might as well write 'Some F is not G'. We prefer the former simply because it is more faithful to the Aristotelian analysis.

Finally some words about the universally accepted procedure of Conversion yielding, for example, the equivalence of 'no F is G' with 'all F is not-G', and of 'some F is G' with 'not all F is not-G'. That is, it has been universally accepted, through the centuries, that external negation may be converted *salva veritate* into internal negation and vice versa, provided *all* is changed into *some* or vice versa. However, as is pointed out by Kneale & Kneale (1962:57), this is not what Aristotle says: 'Aristotle ... allowed that *Every man is not white* could be said to entail *No man is white*, but rejected the converse entailment.' The actual passage in Aristotle (which, moreover, gives some idea of his laborious way of doing logic) is *Int* 20a16-23 (translation mine):

Since the contrary of 'all animals are just' is the statement meaning 'no animal is just', it is clear that these two will never be true at the same time nor be predicated of the same thing. But their contradictories, that is 'not all animals are just' and 'some animal is just', will sometimes be simultaneously true. Then we have the following entailments: 'all men are not-just' entails 'no man is just', and 'some man is just', entails the former's contradictory, 'not all men are not-just'. For in these latter cases there must be at least one man.

The difference is crucial. For, as we shall see in a moment, if the equivalences are given up in favour of one-way entailments from universal to existential sentence types, as Aristotle actually does, his logic is sound: it then no longer suffers from its central logical defect, unwanted existential import. This fact, which to my knowledge has only rarely been observed in the history of logic but never properly elaborated<sup>5</sup>, saves Aristotle from his

<sup>5</sup> According to Kneale & Kneale (1962:210-11), the remarkable Peter Abelard (1079-1144) saw the problem and proposed to reject the equivalence of 'not all F is G' and 'some F is not-G', denying existential import to the former, and also to 'no F is G'. This proposal, however, was not adopted by the later tradition. Nor, apparently, was it properly understood by modern authors on the subject, including William and Martha Kneale.

modern critics on the score of logical soundness, even though what has passed as Aristotelian predicate logic through the centuries does suffer from the defect of undue existential import. In our repair of this traditional ‘Aristotelian’ logic we will actually follow Aristotle in this respect and reject the equivalences in favour of one-way entailments.

At the same time, however, Aristotle assigns, in the last sentence of the quote given above, existential import to ‘not all men are not-just’, and hence to ‘not all men are just’ (since it is immaterial to the logic whether a positive or a negative *G*-predicate is used). This, as we shall see below, is inconsistent with the rejection of the equivalences. His later commentators and interpreters streamlined the system by accepting the equivalences and ignoring Aristotle’s reservations about them. What appears to be the real Aristotelian system (other than in Kneale & Kneale 1962:55) is presented in fig. 3.

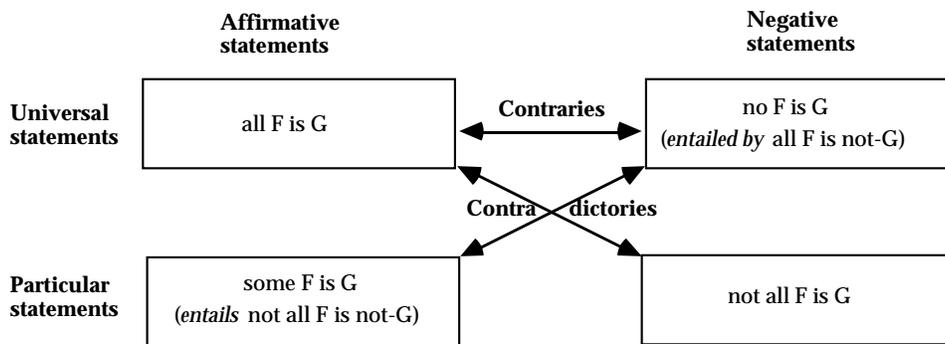


Figure 3. APC as culled from *On Interpretation* and *Prior Analytics*.

ARISTOTELIAN PREDICATE CALCULUS STREAMLINED AND SUMMARIZED BY BOETHIUS

One thing quickly became clear during the centuries following Aristotle. Since the truth of ‘all F is G’ requires the falsity of its contrary ‘no F is G’ and hence the truth of the negation of ‘no F is G’, and since the negation of ‘no F is G’ is its contradictory ‘some F is G’, one must accept that ‘all F is G’ entails ‘some F is G’. And analogously for ‘no F is G’, which is now seen to entail ‘not all F is G’. These two entailment schemata have become

known as the (affirmative and negative) *subalterns*. To what extent Aristotle himself was aware of this consequence of his logic, is not entirely clear, but in his *Prior Analytics* (25a8) he clearly implies the validity of the subaltern entailments (see Kneale & Kneale 1962:58).

Moreover, since the entailments defined for the logical constants *all*, *some* and (external) *not* remain the same whether a predicate *G* or a predicate *not-G* is chosen, the following subaltern entailments must likewise be valid: from ‘all F is not-G’ to ‘some F is not-G’, and from ‘no F is not-G’ to ‘not all F is not-G’.

Acceptance, despite Aristotle’s reservations, of the equivalences created by Conversion as discussed above will obviously simplify and unify the system considerably. This step was taken some time, presumably quite early on (if not already in Aristotle’s own syllogistic), during the eight centuries that lapsed between Aristotle and the Boethian Square.

A final refinement consisted in the discovery that the affirmative and negative particulars, which can be simultaneously true, cannot be simultaneously false. For if they are both false at the same time, their contradictories must both be true at the same time, which is impossible as they are contraries. The pair of particulars ‘some F is G’ and ‘not all F is G’ (now equivalent to ‘not all F is not-G’ and ‘some F is not-G’, respectively) has become known as the pair of *subcontraries*.

This completes the predicate logic as it was summarized by Boethius around the year 500, known to us as *APC*. Boethius, as has been said, introduced the symbols **A** and **I** for the affirmative statement forms ‘all F is G’ and ‘some F is G’, respectively, and the symbols **E** and **O** for the negative statement forms ‘all F is not-G’ and ‘some F is not-G’, respectively, and their alleged equivalents. He cast the entire system into the simple and perspicuous shape of the famous *Square of Oppositions*, which defines the logical relations of the four sentence types with respect to each other, as shown in fig. 4.

To begin with, we see that **A** entails **I** ( $A \vdash I$ ), and **E** entails **O** ( $E \vdash O$ ), i.e. the subaltern entailments. Moreover, **A** and **E** are *Contraries*: they cannot both be true at the same time, though they can be false at the same time. Therefore,  $A \vdash \text{not-E}$  (and thus  $E \vdash \text{not-A}$ ). Then there are the *Subcontraries* **I** and **O**. These can be true at the same time, but not false:  $\text{not-I} \vdash O$  (and thus  $\text{not-O} \vdash I$ ). Finally, there are the *Contradictories*, which cannot be either simultaneously true or simultaneously false. **I** and **E**, and **A** and **O** are pairs of Contradictories:  $I \vdash \text{not-E}$  and  $\text{not-I} \vdash E$ ;  $A \vdash \text{not-O}$  and  $\text{not-A} \vdash O$ . Contradictories may be defined in terms of equivalences. **I** and  $\text{not-E}$  are equivalent, or formally:  $I \equiv \text{not-E}$  (and thus:  $\text{not-I} \equiv E$ ), and  $A \equiv \text{not-O}$  (and thus:  $\text{not-A} \equiv O$ ).

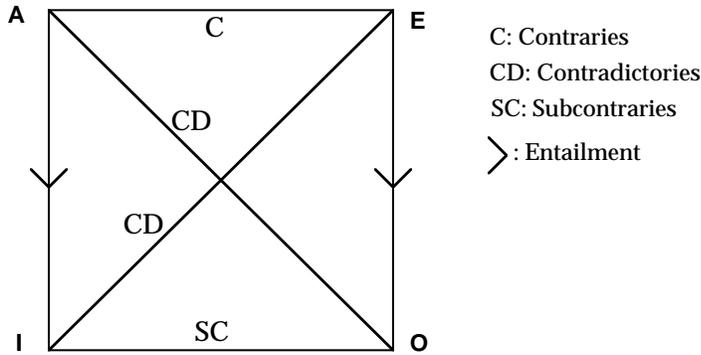


Figure 4. APC as summarized by Boethius in the *Square of Oppositions*.

Finally there is the logical property of *existential import*, not expressed in the Square but encoded in the semantics of the quantifier *some*. For the sentence types I and O it licenses the conclusion that, whenever they are true, there exists at least one member of the *F*-class: there is at least one *F*.

#### THE BOETHIAN REPRESENTATION OF APC IMPROVED

In a moment we will start shooting at  $APC$ . But before we do so, we must first revise the Boethian way of representing  $APC$ , which is far from optimal. It is all very well to use the vowels of the words *affirmo* and *nego* for the four sentence types, but we can do better. One problem with the Boethian Square is that the role of (internal or external) negation is not spelled out explicitly, even though negation does play a systematic structural role in the entailment schemata. Let us, therefore, use the signs ‘ $\neg$ ’ for external negation and ‘ $*$ ’ for internal (clausal predicate) negation. This reduces the number of quantifiers to just *all* and *some*, which are precisely the two that figure in standard modern predicate calculus. The corners of the Square are now named **A**, **I**, **A\*** (or  $\neg$ I), and **I\*** (or  $\neg$ A). And the entailment schemata now look as follows:

- (4) a.  $A \vdash I$                       and         $A^* \vdash I^*$                       (Subalterns)  
       b.  $A \vdash \neg A^*$                   and         $A^* \vdash \neg A$                       (Contraries)  
       c.  $\neg I \vdash I^*$                     and         $\neg I^* \vdash I$                       (Subcontraries)  
       d.  $A \equiv \neg I^*$                     and         $I \equiv \neg A^*$                       (Contradictories)

Moreover, the equivalences by Conversion are not expressed explicitly in the Boethian Square. If we do that, we get a somewhat different configuration, which we will call the *Improved Square*. It is shown in fig. 5, where the pairs of heavy horizontal lines stand for logical equivalence. It must be noted that the actual *logic* has not changed: we are still dealing with  $APC$ . Only the representation has been improved.

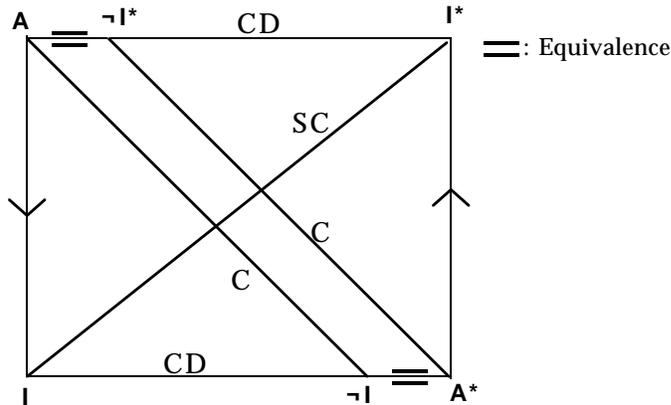


Figure 5. The Improved Square as representation of  $APC$ .

What we have here is, in fact, a combination of two logically analogous triangles, each with the corners  $A$ ,  $I$  and  $\neg I$ , that is, sentence types characterized by the quantifiers *all*, *some* and *no*. The only difference between the two triangles is that the one on the left works with a positive predicate, i.e.  $G$ , whereas the one on the right works with the negative counterpart, i.e. *not-G*. But since the specific choice of predicate has no influence on the logical properties of the calculus, this difference must be logically irrelevant, which it turns out to be: the two triangles display exactly the same logical entailment schemata. Moreover, the triangle on the right hand side stands, so to speak, upside down with respect to the triangle on the left hand side, which enables us to express the equivalences explicitly:  $A \equiv \neg I^*$  and  $\neg I \equiv A^*$ .<sup>6</sup>

<sup>6</sup> The Improved Square also solves a (quasi-)problem raised by Horn (1972, 1989:252-67) and Levinson (2000:69-71), both practitioners of pragmatics. These authors wonder why,

## CRITIQUE OF APC

Now we can start picking holes in APC proper. Around 1900 it became clear that APC is logically faulty: it collapses in cases where the *F*-class is empty. Till that time this had more or less escaped the logicians' notice, though the fact that APC is premised on the assumption of the nonemptiness of the *F*-class was common knowledge in the Middle Ages (see also note 5 above). Consider the sentence:

- (5) All gnomes live in Norway.

This is an impeccable sentence, with the lexical predicate *gnome* in the *F*-position, and the lexical predicate *live in Norway* in the *G*-position. Both predicates are good English, but it so happens that the former is uninstantiated: there are no gnomes in this world, the predicate *gnome* is empty. The question is now: is sentence (5) true or false in this world? According to APC, but also according to modern standard logic, it has to be one of the two. A third possibility is not available ('tertium non datur').

Now suppose we take the sentence to be true. Then it follows, by the subaltern entailment schema, that some gnomes live in Norway (type I). But that again entails, by existential import, that there are gnomes, which is not the case. Therefore, sentence (5) cannot be true. But it cannot be false either. For if it were, it would follow, by the equivalence  $\neg A \equiv I^*$ , that some gnomes do not live in Norway (type I\*), which again has existential import. Therefore, sentence (5) can be neither true nor false, which goes against the very charter of both traditional and modern logic. In fact, the whole of APC turns out to have existential import, as it requires the non-emptiness of the *F*-class. Only non-empty predicates are allowed in the *F*-position.

This is fatal for any logic. For if a logic is, as we have agreed it is, a method for the computation of entailments merely on grounds of the semantic definition of the logical constants, then it must be irrelevant

in the Boethian Square, the A-, I- and E-corners are lexicalized, in virtually all languages of the world, as one single word (in English, for example, as *all*, *some* and *no*), whereas the O-corner systematically requires more than one word as its lexical expression. They seek the answer in pragmatic principles. Although their observation appears to be correct, the Improved Square shows that the problem is imaginary: it is merely an artifact of the deficient way Boethius formalized APC. The Boethian O-corner has lost its place to the three quantifier expressions *all*, *some* and *no*.

whether or not there exist, right now, any instances of  $F$ , be they gnomes or one-hundred-year-old Scots, or black swans.

MODERN PREDICATE CALCULUS (MPC)

Modern predicate calculus has tackled and solved this problem in a radical way. Starting from the Euler diagrams as shown in fig. 1 above, the makers of modern logic simply applied classical set theory, according to which the null set, usually written ' $\emptyset$ ', is a subset of any set: for all sets  $X$ ,  $\emptyset \subseteq X$ . A sentence of the **A**-type is accordingly said to be true in precisely those cases where the  $F$ -class is a subset of the  $G$ -class, whether the former is empty or non-empty. A sentence of type **I** is said to be true in precisely those cases where the  $F$ -class and the  $G$ -class intersect non-emptily: there is at least one  $F$  and at least one  $G$ .<sup>7</sup>

As a result of this, however, virtually the whole of the Square, whether Boethian or Improved, has to go. The Subalterns are no longer valid. Nor are the Contraries, since **A** and **A\*** both count as true when the  $F$ -class is empty. The same fate befalls the Subcontraries, since both **I**-type and **I\***-type sentences count as false when the  $F$ -class is empty. Only the Conversions remain intact. But that is hardly sufficient to speak of a Square of logical relations. In other words,  $_{APC}$  has had to give way to a new predicate calculus which, compared to  $_{APC}$ , is badly impoverished. Yet that new calculus is our  $_{MPC}$ , which was introduced around 1900 and has since been looked up to with the greatest reverence and respect.

This respect is not unjustified. Modern predicate calculus is the most direct, 'minimalist' logical expression of Boolean algebra, and thus of set theory, which is foremost among the foundations of mathematics. This makes  $_{MPC}$  eminently suitable for the formulation of mathematical statements and proofs. Which again means that  $_{MPC}$  is an eminent tool in all applications of mathematics in the sciences and in technology. Countless

<sup>7</sup> While discussing  $_{MPC}$  we disregard the formal *language* of  $_{MPC}$ , which derives from Peano via Russell and Whitehead's *Principia Mathematica*. Since the language of  $_{MPC}$  allows for the possibility of recursive embedding of quantified propositional functions,  $_{MPC}$ 's logical power is vastly superior to that of  $_{APC}$  in its language of the Improved Square as shown in fig. 5. However,  $_{APC}$  is easily reformulated in the language of  $_{MPC}$ , and vice versa, since in the formal language of  $_{MPC}$  the recursive embeddings are themselves always formulated in terms of the original Aristotelian sentence types. For simplicity's sake we restrict ourselves here to representations in the language of the Improved Square. The analysis is in no way affected by this restriction.

20th-century textbooks of logic sing the praises of logic on account of these merits.

But for natural language  $\text{MPC}$  is an unmitigated disaster. Consider, for example, the sentences (6a, b). In the  $\text{MPC}$  book they should both be true in this world, where no real gnomes exist:

- (6) a. All gnomes are fictitious.
- b. No gnome is fictitious.

But we, ordinary mortals, clearly judge (6a) true and (6b) false, just as Aristotelian logic has it, where the two sentences form a pair of Contraries. One may reply that the predicate *fictitious* is intensional, which means that it applies to non-existing entities as well as to existing ones, and that, for that reason, it can be used only in intensional logics and not in purely extensional  $\text{APC}$  OR  $\text{MPC}$ . But even if that is granted, it must be said that the existing formal varieties of intensional logic are of no use in the analysis of language (see Dowty et al. 1981:170-175). Moreover, it is still the case that sentence (5), with the extensional predicate *live in Norway*, is deemed false by natural speakers, as opposed to the sentence (6a), with the intensional predicate *be fictitious*, which is deemed true. We are, therefore, fully entitled to say that existing logic badly fails to account for natural linguistic intuitions.

Consider a second case, this time involving only extensional predicates (note that *gnome* is an extensional predicate, even if there are no gnomes, since to be a real gnome one has to exist). According to  $\text{MPC}$ , both the sentences (7a, b) must count as true in any situation, common enough in the actual world, where there are visitors but no real gnomes:

- (7) a. Some visitors talked with all gnomes.
- b. Some visitors didn't talk with any gnome.

In fact,  $\text{MPC}$  makes both sentences equivalent, given the absence of gnomes, to the statement that there was at least one visitor. Yet ordinary people will consider (7a) false. And (7b) may have to count as true, but only in a trivial and uninformative way. Pragmatic principles make both sentences equally inappropriate in the given context, but fail to explain why the one is felt to be false while the other is taken to be true. More examples of this nature are easily thought up. In practically all cases the conclusion is that  $\text{APC}$  fits natural intuitions much better than  $\text{MPC}$ , even though the latter appears to reign supreme in the world of modern logic.<sup>8</sup>

But we will not give up. After all, we have decided that logic, any logic, is a formal system for the computation of entailments on grounds of the meanings of the logical constants involved. That being so, it should be possible to vary the semantic descriptions of the logical constants concerned while still keeping the logic sound. And if we do that, we may as well keep the presumed logical constants of natural language in mind, trying out, by way of hypothesis, semantic specifications of logical constants to see how well they reflect what is actually going on in language, because that is what we are ultimately interested in.

The strategy we will follow is based on the notion of *valuation space* introduced in Van Fraassen (1971). The valuation space  $/A/$  of a sentence  $A$  is defined as the set of possible situations in which  $A$  is true. These situations are not just 'possible worlds', which have proved to be unmanageable in many ways, but are distinguished in terms of a given set of sentences. The simplest case is a set of one sentence, say  $A$ , which divides the universe  $\mathbf{U}$  of all possible situations up into two subsets, those in which  $A$  is true, or  $/A/$ , and those in which  $A$  is false, or  $/\neg A/$ , where  $/\neg A/$  is the complement of  $/A/$  in  $\mathbf{U}$ .

Valuation space models provide a specific method for the foundation of logical systems, which, however, is hardly used in current logic textbooks. This is regrettable, since valuation space modelling not only makes for a better insight into the principles of logic but it also proves to be of great use when one wishes to develop non-standard variations and show their logical soundness. The present context is a case in point. A valuation space model is, in fact, the optimal graphic representation of  $_{APC}$ ,  $_{MPC}$  and all possible variants. Fig. 6 shows two variants of a valuation space model for  $_{APC}$ .

In fig. 6a the innermost circle is the space for all situations in which the sentence types  $\mathbf{A}$  and  $\mathbf{I}$  are true. The middle ring is the space for all situations in which the types  $\mathbf{I}$  and  $\mathbf{I}^*$  are true. The outer ring contains exactly

<sup>8</sup> During the 1960s  $_{MPC}$  was rightly criticised for being unable to cope with an empty universe of entities. In  $_{MPC}$  the problem of the empty F-class has in fact been pushed to the very edge but has, if one insists, basically remained unsolved. So-called 'free' logics were designed as a remedy (see in particular Lambert 1963). Though of great interest in themselves, these developments are not relevant to the problems discussed in the present context.

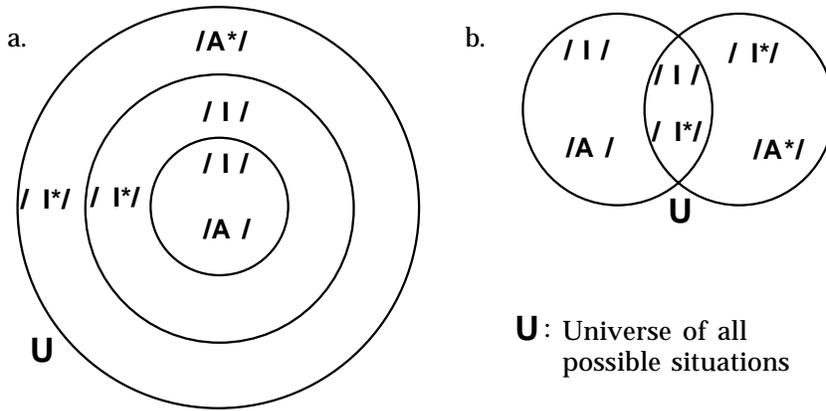


Figure 6. Valuation space models for APC.

those situations where the types  $A^*$  and  $I^*$  are true. Analogous readings for fig. 6b are obvious.

Such diagrams enable one to read entailment schemata directly from them. For all valuation space models it is the case that for each pair of sentences  $X$  and  $Y$ :

- $X \vdash Y$  iff  $/X/$  is a subset of  $/Y/$ , or:  $/X/ \subseteq /Y/$
- CD( $X,Y$ ) iff  $/X/$  is the complement of  $/Y/$ , or:  $/X/ = \overline{/Y/}$
- C( $X,Y$ ) iff  $/X/$  and  $/Y/$  have no element in common, or:  $/X/ \cap /Y/ = \emptyset$
- SC( $X,Y$ ) iff  $/X/$  and  $/Y/$  together form  $U$ , or:  $/X/ \cup /Y/ = U$
- $\equiv (X,Y)$  iff  $/X/$  and  $/Y/$  are identical, or:  $/X/ = /Y/$ .

The entire logical system of APC can now be read from figs. 6a or 6b. The Subaltern entailments are valid because  $/A/$  is a subset of  $/I/$  and  $/A^*/$  is a subset of  $/I^*/$ . The Contradictories (CD) are valid because  $/I/$  is the complement of  $/A^*/$  and  $/A/$  is the complement of  $/I^*/$ . This means that the Equivalences (Conversions) are valid:  $/A/ = \neg/I^*/$  and  $/I/ = \neg/A^*/$ . The Contraries (C) are valid because  $/A/$  and  $/A^*/$  have no element in common. The Subcontraries (SC), finally, are valid because  $/I/$  and  $/I^*/$  together form  $U$ .

The problem is, however, that the space  $U$  of figs. 6a or 6b does not contain all possible situations with respect to the sentence types involved, because the situations where the  $F$ -class is uninstantiated are not accounted for. But now that we have a valuation space model at our disposal, the

remedy is ready at hand: we simply add a further ring to fig. 6a or 6b for those situations where the *F*-class is empty. The result is figs. 7a or 7b, which show a valuation space modelling for what we will call the *revised Aristotelian predicate calculus* or *RAPC*.

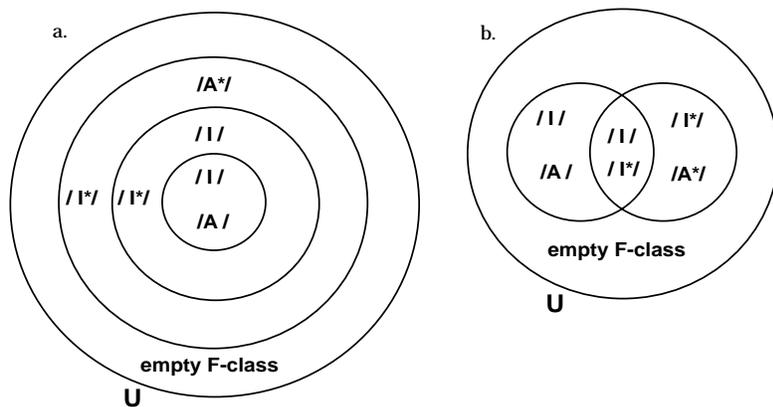


Figure 7. Valuation space models for *RAPC*.

Now *U* contains all possible situations with respect to the four sentence types and their negations. But at the same time the addition of the outer ring in figs. 7a,b has changed the entailment schemata, as one can see directly by inspecting the diagrams of figs. 7a,b. The Subalterns have remained intact, and also the Contraries, whose number has even increased. But the Subcontraries have disappeared, and so have the Equivalences, which have been replaced with one-way entailments:  $A \vdash \neg I^*$  en  $A^* \vdash \neg I$ , but not vice versa. Therefore, the Improved Square of fig. 5 must be replaced with a different representation, shown in fig. 8, which gives the *Hexagon of Logical Relations* in *RAPC*.

What fig. 7 does but fig. 8 does not show is the fact that in *RAPC*, when the *F*-class is empty, all four sentence types *A*, *I*, *A\** and *I\** count as false. The difference with *MPC* is that *MPC* counts the types *A* en *A\** as true in such cases. This again means that the entailment schemata of *MPC* differ significantly from those in *RAPC*, as one can see from fig. 9. Perhaps surprisingly, the only difference between the valuation space models for *MPC* and for *RAPC* consists in the fact that in *MPC* *A* en *A\** count as true with an empty *F*-class, whereas in *RAPC* they count as false.



Our answer is that they are likely to be productive, pending results of psycholinguistic experiments. The loss of the Subcontraries probably is an advantage, since a sentence like (8), which instantiates the simultaneous falsity of  $I$  and  $I^*$ , appears appropriate in a situation without any gnomes:

- (8) I didn't see any gnome there, nor was there any gnome I did not see.

In this respect we have to admit that both  $_{MPC}$  and  $_{RAPC}$  are superior to  $_{APC}$ . But for the Equivalences the situation is different. It does not seem to be absurd to regard a sentence like (9a) as true while taking (9b) to be false, although  $_{MPC}$  deems them both true:

- (9) a. There was no gnome in the room.  
b. All gnomes were not in the room (outside the room).

It does, moreover, seem reasonable to suppose that for natural intuition (9b) entails (9a), or  $9b \vdash 9a$ , but not vice versa, which would favour  $_{RAPC}$  over both  $_{APC}$  and  $_{MPC}$ . On the other hand, it looks as if sentences like (10a) and (10b), which are likewise both considered true in  $_{MPC}$ , count as false for natural intuition:

- (10) a. No gnome said anything.  
b. All gnomes said nothing.

Moreover, one has the feeling that not only  $10a \vdash 10b$  but also  $10b \vdash 10a$ , which would make (10a) equivalent with (10b). If that is correct,  $_{RAPC}$  is to be rejected in favour of  $_{MPC}$  (or  $_{APC}$  if that were a sound system).

If experimental evidence were to corroborate these provisional intuitions, we would be in a logico-semantic quandary, as several systems would then appear to co-exist side by side. Perhaps we should think of two distinct existential quantifiers in natural language, with partly different realisations in surface structure. Negation of the one would then yield truth in case of an empty  $F$ -class, whereas negation of the other would yield falsity. In view of what is proposed in the following section this may well be an interesting thought.

#### MORE POSSIBILITIES

But we can go further than this. Since Strawson (1950) it has been widely accepted that the normal, unmarked negation in natural language

negates only part of the semantic content of the sentence while leaving the remainder unaffected. The unaffected remainder is what we call the *presuppositions* of the sentence; the negated part is the *assertive content*. A sentence like (11a), for example, *presupposes* that John wants to go by bike and *asserts* that John is the only one who wants that. But (11b), though the negation of (11a), still presupposes that John wants to go by bike, although now the assertion that he is the only one has been denied:

- (11) a. Only John wants to go by bike.  
 b. Not only John wants to go by bike.

In other words, the negation word *not* denies only the *assertion* of (11a) but leaves the *presupposition* intact.

If this analysis is correct, it follows that the standard logical negation does not represent normal, unmarked negation in natural language. While the standard logical negation, represented as  $\neg$ , corresponds with the valuation space diagram of fig. 10a, natural language negation appears to be represented more adequately as in fig. 10b, with the sign ' $\sim$ ' for presupposition-preserving natural language *not*. The heavy circle in fig. 10b contains the situations where the presuppositions of a sentence *A* are true. The middle ring is the so-called *inner complement* of  $/A/$ , i.e. the set of situations where the presuppositions of *A* are true but not the assertive content of *A*. The outer ring is the *outer complement* of  $/A/$ , i.e. the set of situations where the presuppositions of *A* are false.

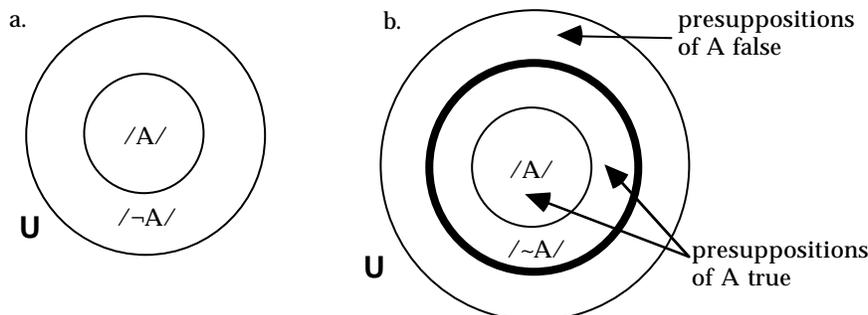


Figure 10. Valuation space diagrams for logical and linguistic negation.



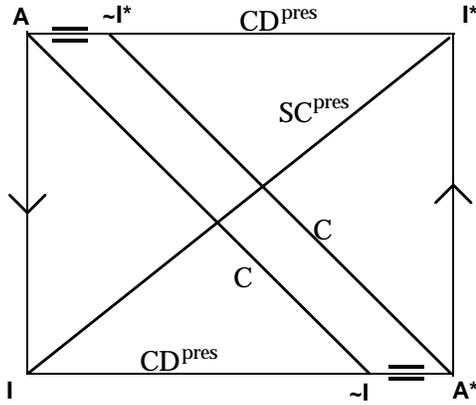


Figure 12. Improved Square with presupposition-preserving negation and non-empty  $F$ -class presupposition for  $A$ - and  $I$ -type sentences.

This leads to a mixed system, whose valuation space model is given as fig. 13.

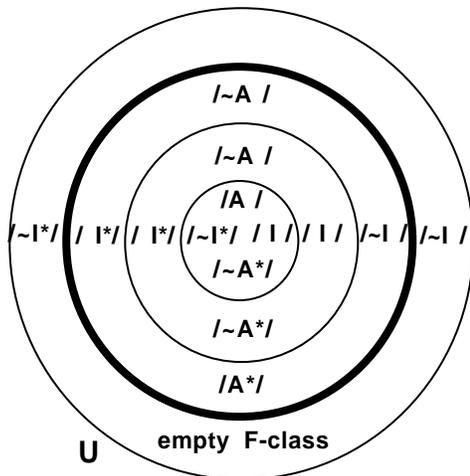


Figure 13. Valuation space model for  $RAPC$  with presupposition-preserving negation and non-empty  $F$ -class presupposition only for  $A$ -type sentences.

Fig. 13 returns the Hexagon of Logical Relations of fig. 8, but now, as shown in fig. 14, with the presupposition-preserving negation ' $\sim$ ' instead of ' $\neg$ '.

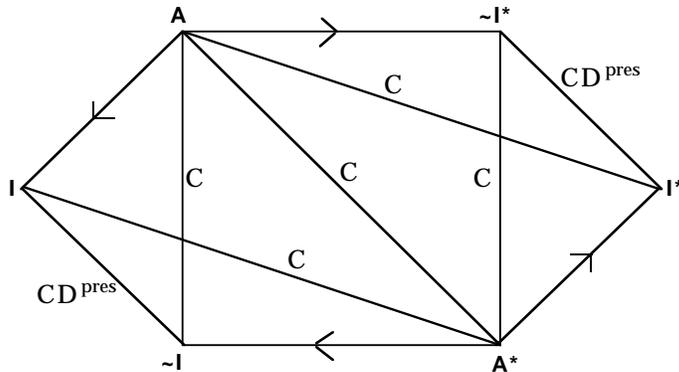


Figure 14. Hexagon with presupposition-preserving negation and non-empty  $F$ -class presupposition only for  $A$ -type sentences.

Special care, however, must be taken, in this mixed system, with regard to entailment relations between  $A$ -type and  $I$ -type sentences. The definitions of entailment, contradictoriness, contrariety, subcontrariety and equivalence given above still stand, but they no longer simply translate into entailment schemata. Entailments from  $\sim I$ - or  $\sim I^*$ -type to  $A$ - or  $A^*$ -type sentences with or without the negation  $\sim$ , are, on the whole, no longer valid. To see which are and which are not valid, one only has to consult fig. 13, which allows one to read all entailment relations directly. Interestingly, though  $I$  and  $A^*$  in fig. 14 are still Contraries, they are now also Contradictories within the inner complement for the  $A$ -type sentences.

The systems of fig. 11 and fig. 13 may be valid at the same time, if we accept two different existential quantifiers, one with and one without the non-empty  $F$ -class presupposition, as was suggested above, at the end of the previous section. In other words: options galore. But the question is: which of these options, or which combination of options, provides the most adequate account of the natural intuitions of native speakers in terms of a sound logic. The dilemma that beset philosophers of language and semanticists a century ago when modern logic was introduced may, after all, turn out less serious than has been thought.

WHY?

In the given context one is naturally inclined to pose the obvious question of the functionality of the specific variety or varieties of logic presumed to be present in language and thinking. Why do language and thinking deviate from the minimalist logic system that has been found adequate for mathematical purposes?

This question, however, no matter how justified, cannot be answered at this stage. This is because the prior question as to the *actual nature* of the logic of language and thinking has not been answered yet. The first large problem still awaiting a solution is posed by quantification involving intensional predicates and the closely related issue of quantification over intensional entities. This problem area, touched upon above in connection with the examples (6a, b), is of considerable magnitude and a satisfactory account will no doubt necessitate drastic measures with regard to the existing machinery. Sentence (12), for example, shows that when an intensional predicate like *be worshipped* is used in the *G*-position, an *I*-type sentence can still be true even though the *F*-class is uninstantiated:

(12) A Greek god was worshipped there in the old days.

Such cases seem to warrant the conclusion that in predicate calculus as it occurs in language and thinking the existential quantifier does not have existential import (only ontological import), existential import being derived from the extensional character of extensional predicates. Such matters, however, are far from trivial.

It seems, moreover, that any investigation into questions of the functionality of the logical system or systems postulated for language and cognition had better be carried out in the context of certain hypotheses concerning the general nature of cognition. At this moment we have two specific hypotheses in mind. First we would consider the so-called *null set hypothesis*, which says that the concept 'null set' does not occur in natural human thinking. If this hypothesis is correct, the logic of language must be represented without the help of the symbol  $\emptyset$  — a task at which we are working. The second hypothesis is the *proper subset hypothesis*, which says that in natural human thinking the notion 'subset' is always interpreted as 'proper subset', just as a subcontinent is necessarily a part of a larger area called 'continent', or a subcontract is part of a large contract. These, however, are perspectives for further research about which we cannot report at this stage.

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